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SOMPATY

Conference
on
Mathematics for the Micro/Nano-World:
From soliton dynamics, nonlinear optics to quantum science and technology

Program & Book of Abstracts

September 18-22, 2023
Samarkand, Uzbekistan

TECHNISCHE UNIVERSITÄT ILMENAU

TURIN POLYTECHNIC UNIVERSITY IN TASHKENT

NUCLEAR PHYSICS INSTITUTE CAS
public research institution
The Czech Academy of Sciences

UNIVERSITÄT ZU LÜBECK

Scientific Program

INTERNATIONAL SUMMER SCHOOL

on

Mathematics for the Micro/Nano-World: From soliton dynamics, nonlinear optics to quantum science and technology,

September 11 – 16, 2023
Samarkand, Uzbekistan

Sponsored

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<u>September 11</u>

Arrival of participants.
Welcome reception (at 18:30) and dinner.

<u>Day 1, September 12</u>

08:30-09:50 *Registration*

09:50 –10:00 *Opening*

Morning Session
Chair: Carsten Trunk

10:00-11:00 Pavel Exner. *Guided quantum dynamics*

11:00 -11:15 Questions and discussions

Coffee Break (11:15-11:45)

11:45-12:45 Pavel Exner. *Guided quantum dynamics*

12:45-13:00 Questions and discussions

Lunch break (13:00-15:00)

Afternoon Session
Chair: Pavel Exner

15:00-16:00 Olaf Post. *Analysis on graph-like spaces*

16:00-16:15 Questions and discussions

Coffee Break (16:15-16:45)

16:45-17:45 Olaf Post. *Analysis on graph-like spaces*

17:45-18:00 Questions and discussions

Dinner (19:00-21:00)

<u>Day 2, September 13</u>

Morning Session
Chair: Juergen Prestin

10:00-11:00 Pavel Exner. *Guided quantum dynamics*

11:00-11:15 Questions and discussions

Coffee Break (11:15-11:45)

11:45-12:45 Pavel Exner. *Guided quantum dynamics*

12:45-13:00 Questions and discussions

Lunch break (13:00-15:00)

Afternoon Session
Chair: Jaroslav Dittrich

15:00-16:00 Olaf Post. *Analysis on graph-like spaces*

16:00-16:15 Questions and discussions

Coffee Break (16:15-16:45)

16:45-17:45 Olaf Post. *Analysis on graph-like spaces*

17:45-18:00 Questions and discussions

Dinner (19:00-21:00)

Day 3, September 14

Morning Session
Chair: Riccardo Adami

10:00-11:00 Pavel Exner. *Guided quantum dynamics*

11:00-11:15 Questions and discussions

Coffee Break (11:15-11:45)

11:45-12:45 Olaf Post. *Analysis on graph-like spaces*

12:45-13:00 Questions and discussions

Lunch break (13:00-15:00)

Afternoon Session
Chair: Olaf Post

15:00-16:00 Riccardo Adami. *Nonlinear Schroedinger Equation on exotic domains*

16:00-16:15 Questions and discussions

Coffee Break (16:15-16:45)

16:45-17:45 Riccardo Adami. *Nonlinear Schroedinger Equation on exotic domains*

17:45-18:00 Questions and discussions

Conference Dinner (19:00-22:00)

<u>Day 4, September 15</u>

Morning Session
Chair: Henrik Winkler

10:00-11:00 Riccardo Adami. *Nonlinear Schroedinger Equation on exotic domains*

11:00-11:15 Questions and discussions

Coffee Break (11:15-11:45)

11:45-12:45 Riccardo Adami. *Nonlinear Schroedinger Equation on exotic domains*

12:45-13:00 Questions and discussions

Lunch break (13:00-15:00)

Afternoon Session
Chair: Davron Matrasulov

15:00-16:00 Riccardo Adami. *Nonlinear Schroedinger Equation on exotic domains*

Dinner (19:00-21:00)

Day 5, September 16

Morning (Online) Session
Chair: Andreas Roessler

10:00-11:00 Delio Mugnolo. *Diffusion problems on metric graphs*

11:00-11:15 Questions and discussions

Coffee Break (11:15-11:45)

11:45-12:45 Delio Mugnolo. *Diffusion problems on metric graphs*

12:45-13:00 Questions and discussions

Lunch break (13:00-15:00)

Afternoon Session
Chair: Jambul Yusupov

15:00-16:00 Delio Mugnolo. *Diffusion problems on metric graphs*

16:00-16:15 Questions and discussions

Dinner (19:00-21:00)

Day 6, September 17

10:00-13:00 Guided excursion in Samarkand

Lunch break (13:00-15:00)

15:00-16:00 Academic free discussions

16:00-16:30 Discussions

Closing the school

INTERNATIONAL SUMMER SCHOOL

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Abstracts of Lectures

Guided quantum dynamics

Pavel Exner

Doppler Institute for Mathematical Physics and Applied Mathematics, Prague

This minicourse is devoted to the quantum graphs and waveguides, that is, quantum dynamics in the situation when the motion is localized in one direction while in the other(s) the system is allowed to propagate. We consider different types of the transverse localization, a hard wall represented by Dirichlet condition, or trapping by a potential, a regular or singular one. We are concerned primarily in relations between the geometry and topology of the confinement on the one hand, and the spectral and transport properties of these systems on the other.

A brief overview of the minicourse:

Lecture I: Quantum graphs, where they come from and what they are good for. Resonances and spectral gaps
Lecture II: Quantum waveguides and layers. Spectral and scattering properties coming from their geometry
Lecture III: Taking quantum tunneling into account: leaky graphs and soft waveguides
Lecture IV: Graphs violating the time-reversal invariance, and what that means for their spectral and transport properties
Lecture V: Spectral optimization of graphs and waveguides. Effects of magnetic fields. Summary and outlook

Analysis on graph-like spaces

Olaf Post

University of Trier, Germany

In this mini-course I will give some basic spectral analysis on spaces that resemble graphs, including discrete graphs, metric graphs and spaces that have a network-like structure. As a simple example of such a structure one can think of a small neighbourhood of a metric graph embedded in the plane. A natural question is now whether and how Laplacians (or more general Schrödinger operators) on the small neighbourhood converge to some limit operator on the underlying metric graph.

I will develop the abstract ideas along many examples in order that students with different background are able to follow. A rough structure of my lectures is as follows:

1. Fixing the notation: Discrete and metric graphs and their Laplacians
2. Spectra of discrete and metric graphs
3. Graph-like spaces: thin neighbourhoods of graphs
4. The tool-box: how to define convergence of operators when the spaces change?
5. The results: convergence of Laplacians on graph-like spaces

Nonlinear Schroedinger Equation on exotic domains

Riccardo Adami

Turin polytechnic university (Italy)

In this short course I will present some aspects of the problem of the existence of Ground States for the Nonlinear Schroedinger Equation on exotic domains, like metric graphs and hybrids, namely, domains constructed by glueing together component of different space dimensions. I will follow this scheme:

1. Introduction: atomtronic and NLS on exotic structures. The definition and the meaning of a Ground State.

2. The role of topology of graphs, and a sufficient condition for the non existence of Ground States

3. The role of the metric of graphs in the existence and nonexistence. Examples.

4. The case of the critical nonlinear Schroedinger Equation. Why and how all previous intuitions fail.

5. The case of a simple hybrid.

Diffusion problems on metric graphs

Delio Mugnolo

University of Hagen (Germany)

In this mini-lecture I will present the main features of linear diffusion equations on metric graphs, including a gentle introduction to heat kernels and some distinguished qualitative properties, especially smoothness and pointwise bounds.

I will show how variational methods can be used to estimate the speed of convergence towards equilibrium of solutions, as well as the effectiveness of thermal insulation. I will present some elementary results in shape optimization of metric graphs with respect to both quantities.

If time allows, I will also mention some possible extensions to nonlinear diffusion.

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Day 1, September 18

08:30-09:50	*Registration*
09:50 –10:00	*Opening*

Morning Session
Chair: Carsten Trunk

10:00-10:40	Pavel Exner. *Curvature-induced discrete spectrum in soft quantum waveguides and dot arrays*
10:40-11:20	Delio Mugnolo. *Spectral and Torsion Geometry of Quantum Graphs*

Coffee Break (11:20-11:50)

11:50-12:30	Olaf Post. *Norm resolvent convergence of discretisations of the Laplacian on domains*
12:30-13:10	Jiří Lipovský. *Magnetic quantum graphs with preferred-orientation coupling*

Lunch break (13:10-15:00)

Afternoon Session
Chair: Pavel Exner

15:00-15:40	Carsten Trunk. *Spectral optimization for algebraic differential equations with application to chip desi*
15:40-16:20	Mitsuru Wilson. *Spectral inclusion property for a class of block operator matrices*

Coffee Break (16:20-16:50)

16:50-17:30	Henrik Winkler. *Jordan-like decompositions of linear relations and DAEs*
17:30-18:10	Mersaid Aripov. *Role of the critical exponents in the problem nonlinear heat conductivity with variable density and time dependent damping*
18:10-18:30	Narkesh Iskakova. *On the solvability of the periodic boundary value problem for delay differential equation*
18:30-18:50	Zhanibek Tokmurzin. *Method of functional parametrization for solving a semi-periodic initial problem for fourth-order partial differential equations*

Dinner (19:00-21:00)

Day 2, September 19

Morning Session
Chair: Jurgen Prestin

10:00-10:40	Jambul Yusupov. *Transparent boundary conditions for nonlinear evolution equations in low-dimensional domains*
10:40-11:20	Zarif Sobirov. *Inverse source problems with integral overdetermination for the subdiffusion equations on metric graphs*

Coffee Break (11:20-11:50)

11:50-12:30	Zhazira Kadirbayeva. *A computational method for solving problem for impulsive differential equations with loadings*
12:30-12:50	Altynshash Bekbauova. *Creation of decisions in a broad sense the systems of the equations in private derivatives of the first order with periodic conditions*

Lunch break (13:00-15:00)

Afternoon Session
Chair: Jambul Yusupov

15:00-15:40	Roza Uteshova. *Singular boundary value problems for ODEs and their approximation*
15:40-16:20	Gulmira Vassilina. *Construction of the Lagrange equation by a given kinematic form of the Newton equation in the presence of random perturbations*

Coffee Break (16:20-16:50)

16:50-17:30	Anar Assanova. *Boundary value problems for partial differential equations with impulse discrete memory*
17:30-18:10	Mashrab Akramov. *Transparent boundary conditions for nonlinear nonocal Schrodinger equation on metric graphs*
18:10-18:30	Symbat Kabdrakhova. *A modification of the Euler polygonal method for solving semi-periodical boundary value problem for loaded nonlinear hyperbolic equation*

Conference Dinner (19:00-22:00)

Day 3, September 20

Morning Session
Chair: Olaf Post

10:00-10:40	Sandugash Mynbayeva. *One approach to find an approximate and a numerical solution of a nonlinear boundary value problem*
10:40-11:20	Jaroslav Dittrich. *Scattering along a curve in a plain*

Coffee Break (11:20-11:50)

11:50-12:30	Umida Baltaeva. *On the solvability of problems for the loaded integro-differential equation involving a Riemann–Liouville fractional integral operator*
12:30-12:50	Miloš Tater. *In search of a higher Bochner Theorem*

Lunch break (13:00-15:00)

Afternoon Session
Chair: Jaroslav Dittrich

15:00-15:40	Davron Matrasulov. *Relativistic quasiparticles on metric graphs*
15:40-16:20	Gisela Poplau. *The Numerical Solution of the 3D-Poisson Equation for Applications in Accelerator Physics*

Coffee Break (16:20-16:50)

16:50-17:30	Jurgen Prestin. *Shift-invariant spaces of periodic functions*
17:30-18:10	Lena Schadow. *The Stochastic Wave Equation – Why and How to Model Random Fluctuations*
18:10-18:30	David Spitzkopf. *Tunneling in soft waveguides: closing a book*

Dinner (19:00-21:00)

Day 4, September 21

Morning Session
Chair: Delio Mugnolo

10:00-10:40	Oleksandra Antoniouk. *Spectral methods in the theory of p-adic nonlinear nonlocal pseudo-differential equations*
10:40-11:20	Elchin I. Jafarov. *Slightly modified orthogonality relation for the Jacobi polynomials and exactly solvable model of the completely positive oscillator-shaped quantum well*

Coffee Break (11:20-11:50)

11:50-12:30	Saparboy Rakhmanov. *Quantum Fermi acceleration under driven time-dependent confinement*
12:30-12:50	Lutfiya Rajabova. *To the theory of the overdeterminated system of Volterra-type three integral equations with singular boundary lines*

Lunch break (13:00-15:00)

Afternoon Session

15:00-15:20	Danylo Yakymenko. *The order 3 symmetry in the Weyl-Heisenberg group.*
15:20-15:40	Zahriddin Muminov. *On the discrete spectrum of the Schrodinger operator of the system of three-particles with masses $m_1=m_2=\infty$ and $m_3< \infty$ on the three dimensional lattice*
15:40-16:00	Kuvonchbek Matchonov. *Overcritical spectrum of the one-dimensional Dirac equationin bulk and confined domains*
16:00-16-20	***Discussions***
16:20-16:40	***Closing conference***

Day 5, September 22

Departure of participants

CONFERENCE

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BOOK

of Abstracts

CURVATURE-INDUCED DISCRETE SPECTRUM IN SOFT QUANTUM WAVEGUIDES AND DOT ARRAYS

Pavel Exner

Doppler Institute for Mathematical Physics and Applied Mathematics, Prague

We consider Schrödinger operators with a regular attractive potential extended in one direction, a potential ‘ditch’ or an array of potential wells, and investigate the discrete spectrum which arises if the potential support is not straight. Furthermore, we will discuss ground-state optimalization in case of loop-shaped configurations; a few related problems will be also mentioned.

NORM RESOLVENT CONVERGENCE OF DISCRETISATIONS OF THE LAPLACIAN ON DOMAINS

Olaf Post
University of Trier, Germany

In this talk, I will discuss some new results of norm resolvent convergence of discretisations of Laplacians on Euclidean domains. The main idea is to use certain triangulations and suitable splines for the discretization. We completely avoid using Fourier methods.

MAGNETIC QUANTUM GRAPHS WITH PREFERRED-ORIENTATION COUPLING

Lipovský Jiří
University of Hradec Kralove, Czech Republic

One advantage of the model of quantum graphs is that various self-adjoint coupling conditions can be imposed at the vertices. One possibility, originally suggested by P. Exner and M. Tater, is the preferred-orientation coupling condition. We will briefly revise its main properties, previously introduced in one of prof. Exner's talk in the summer school. We will focus on the behaviour of the model under the influence of the magnetic field. We discuss the properties of two models - the infinite ring chains and the magnetic square lattice with this coupling condition under the influence of the magnetic field. We focus on the spectrum, for the former model we discuss e.g. the Band-Berkolaiko universality of the spectrum. For the latter model, we show that Hofstadter's butterfly pattern appears when the spectrum for the rational flux ratio per plaquette is displayed.

The talk will be based mainly on the following two papers obtained jointly with M. Baradaran and P. Exner.

[1] M. Baradaran, P. Exner, J. Lipovský, Magnetic ring chains with vertex coupling of a preferred orientation, J. Phys. A: Math. Theor. 55 (2022), 375203.
[2] M. Baradaran, P. Exner, J. Lipovský, Magnetic square lattice with vertex coupling of a preferred orientation, Ann. Phys. 454 (2023), 169339.

TRANSPARENT BOUNDARY CONDITIONS FOR NONLINEAR EVOLUTION EQUATIONS IN LOW-DIMENSIONAL DOMAINS

Jambul Yusupov
Kimyo International university in Tashkent, Uzbekistan

Many physical problems described in terms of a PDE which is defined on an unbounded domain are usually solved numerically by restricting the computational domain by introducing artificial boundary conditions. These artificial boundary conditions are constructed with the objective to approximate the exact solution of the whole-space problem, restricted to the interval. Such boundary conditions are called transparent boundary conditions. The approach has found its application in development of effective models providing tools for tunable wave transport in low-dimensional domains. Here we consider the problem of the absence of backscattering in the transport of solitons in low-dimensional networks modeled in terms of metric graphs. This approach allows to derive simple constraints, which link the equivalent usual Kirchhoff-type vertex conditions to the transparent ones.

QUANTUM FERMI ACCELERATION UNDER DRIVEN TIME-DEPENDENT CONFINEMENT

S. Rakhmanov[a], C. Trunk[b] and D. Matrasulov[c]
[a]Chirchik State Pedagogical University, 104 Amur Temur Str., Chirchik, 111700, Uzbekistan
[b]Technical University of Ilmenau, 25 Weimarer Str., 98693, Ilmenau, Germany
[c]Turin Polytechnic University in Tashkent, 17 Niyazov Str., 100095, Tashkent, Uzbekistan

Despite the fact that considerable aspects of the problem of dynamical confinement have been considered, some issues in the topic are still remaining as less- or not studied. This concerns such aspects as time-dependent Neumann boundary conditions, non-adiabatic limit and exactly solvable models. Another important problem in this context is extension of the model to the case when time-dependent box interacts with an external potential. In such case, if the potential is position independent, the problem approves factorization of space and time variables. In this talk, we will present new results on quantum dynamics of a particle confined in a box with time-dependent wall is revisited by considering some unexplored aspects of the problem. In particular, the case of dynamical confinement in a time-dependent box in the presence of purely time-varying external potential is treated by obtaining exact solution. Also, some external potentials approving separation of space and time variables in the Schrödinger equation with time-dependent boundary conditions are classified. Time-dependence of the average kinetic energy and average quantum force are analyzed. A model for optical high harmonic generation in the presence of dynamical confinement and external linearly polarized monochromatic field is proposed.

SUPERCRITICAL PHENOMENA ON THE RELATIVISTIC ONE-DIMENSIONAL HYDROGEN-LIKE ATOMS

K. Matchonov[a], S. Rakhmanov[b] and D. Matrasulov[c]
[a]National University of Uzbekistan, 4 University Str., Tashkent, 100174, Uzbekistan
[b]Chirchik State Pedagogical University, 104 Amur Temur Str., Chirchik, 111700, Uzbekistan

[c]Turin Polytechnic University in Tashkent, 17 Niyazov Str., 100095, Tashkent, Uzbekistan

Quantum mechanics of a relativistic one-dimensional hydrogen atom is studied by considering two cases: bulk (unconfined) atom and an atom confined in a 1D box. The system is described by the one-dimensional Dirac equation with Coulomb potential. Energy spectrum and local density of states are analysed by solving Dirac equation numerically. Also, critical charge for bulk and confined 1D relativistic Coulomb problem for Dirac equation is calculated by taking into account finite size of the atomic nucleus.

TRANSPARENT BOUNDARY CONDITIONS FOR NONLOCAL NONLINEAR SCHRÖDINGER EQUATION ON METRIC GRAPHS

M.E. Akramov[1], J.R. Yusupov[2], M. Ehrhardt[3] and D.U. Matrasulov[4]

[1]National University of Uzbekistan, Universitet Str. 4, 100174, Tashkent, Uzbekistan

[2]Kimyo International University in Tashkent, 156 Usman Nasyr Str., 100121, Tashkent, Uzbekistan

[3]Bergische Universität Wuppertal, Gaußstrasse 20, D-42119 Wuppertal, Germany

[4]Turin Polytechnic University in Tashkent, 17 Niyazov Str., 100095, Tashkent, Uzbekistan

Transparent boundary conditions have been derived for different PDEs [1-3]. For the nonlocal nonlinear Schrödinger (NNLS) equation on a line they have been derived in a recent work [3], in which a very effective method called potential approach was used. Such boundary conditions allow to obtain the solution of an initial value problem given in an interval, which coincides with the solution of the problem for the whole space confined in this interval. Inspiring by the results, in this work we extend the idea to the branched domains which are modeled by means of metric graphs. We consider NNLS equation on the star graph with four bonds $b_{\pm j}$ ($j = 1,2$), for which coordinates $x_{\pm j}$ are assigned. We choose the origin of coordinates at the central vertex, so that bond b_{-j} will take values $x_{-j} \in (-\infty, 0]$ and for b_j we fix $x_j \in [0, +\infty)$:

$$i\partial_t q_{\pm j}(x,t) + \partial_x^2 q_{\pm j}(x,t) + \sqrt{\beta_j \beta_{-j}}\, q_{\pm j}^2(x,t) q_{\mp j}^*(-x,t) = 0, \tag{1}$$

where $q_{\pm j}(x,t)$ are defined in $x \in b_{\pm j}$, and $j = 1,2$. We choose vertex boundary conditions proposed in Ref. [4]:

$$\gamma_1 q_1(x,t)|_{x=0} = \gamma_{-1} q_{-1}(x,t)|_{x=0} = \gamma_2 q_2(x,t)|_{x=0} = \gamma_{-2} q_{-2}(x,t)|_{x=0},$$
$$\gamma_1 \partial_x q_1(x,t)|_{x=0} + \gamma_2 \partial_x q_2(x,t)|_{x=0} = \gamma_{-1} \partial_x q_{-1}(x,t)|_{x=0} + \gamma_{-2} \partial_x q_{-2}(x,t)|_{x=0}. \tag{2}$$

Considering generalized Kirchhoff boundary conditions (2) and using above mentioned potential approach we derived condition in the form of sum rule

$$\frac{1}{\beta_1} + \frac{1}{\beta_2} = \frac{1}{\beta_{-1}} + \frac{1}{\beta_{-2}} \tag{3}$$

satisfaction of which implies that the vertex boundary conditions (2) become equivalent to the TBC at the vertex. The confirmation of the reflectionless transition through the vertex was achieved by the simulation of traveling solitons. Finally, we note that the proposed model can be used for the development and design of PT-symmetric optical waveguide networks that allow reflectionless propagation of solitons. The practical application of such functional materials in optoelectronic devices would allow to save resources and improve performance by reducing signal losses. This can be done by determining physically relevant conditions for the transparency of the branching points of the network.

Acknowledgement: The work is supported by the grant of the Ministry for Innovation Development of Uzbekistan (Ref. No. F-2021-440).

References:

1. X. Antoine, A. Arnold, C. Besse, M. Ehrhardt, and A. Schädle, Commun. Comput. Phys., **4(4)**, 729 (2008).
2. M. Ehrhardt, Numer. Math. Theor. Meth. Appl. **3**, pp. 295-337 (2010).
3. M.E. Akramov, J.R. Yusupov, M. Ehrhardt, H. Susanto, D.U. Matrasulov, Phys. Lett. A **459**, 128611 (2023).
4. M. Akramov, K. Sabirov, D. Matrasulov, H. Susanto, S. Usanov, and O. Karpova, Phys. Rev. E **105**, 054205 (2022).

SPECTRAL OPTIMIZATION FOR ALGEBRAIC DIFFERENTIAL EQUATIONS WITH APPLICATION TO CHIP DESIGN

Carsten Trunk
Ilmenau Technical University, Germany

Optimization for chip design problems allow a reformulation as a perturbation problem for algebraic differential equations. After Laplace transformation, an algebraic differential equation $E\dot{x} = Ax$ turns into a spectral problem for the pencil

$$\lambda E - A,$$

where E and A are either matrices or operators. We show how such pencils are related to linear subspaces of the form

$$\ker\,[A \ - \ E] \quad \text{and} \quad \operatorname{ran}\begin{bmatrix} A \\ -E \end{bmatrix}.$$

The idea is to take advantage of the well-developed spectral theory of subspaces (or, what is the same, of linear relations). The above relation is used to study various spectral properties of pencils like behaviour under rank one perturbations, distance to singularity, etc. Moreover, it allows a simple approach to infinite dimensions, i.e., when A and B are bounded or unbounded operators, e.g., invariance of the essential spectrum under perturbations.

In collaboration with T. Berger (Paderborn), H. Gernandt (Cottbus), F. Martinez Peria (La Plata), D. Krauße, F. Philipp, R. Sommer, H. Winkler (all Ilmenau), and M. Wojtylak (Krakow)

JORDAN-LIKE DECOMPOSITIONS OF LINEAR RELATIONS AND DAES

Henrik Winkler
joint work with T. Berger, H. de Snoo and C.Trunk
Ilmenau Technical university (Germany)

It is shown that any linear relation (or multi-valued linear transformation) in a finite-dimensional space has a reducing sum decomposition consisting of at most three different types of linear relations:

1. *Multishifts*: injective relations without eigenvalues;

2. *Jordan relations*: relations with a finite number of eigenvalues (including possibly ∞), which are made up of the corresponding Jordan chains;

3. *Singular relations*: multi-valued relations which are made up of so-called singular chains; each complex number is an eigenvalue.

Connections to the solution theory of DAEs are presented.

SPECTRAL INCLUSION PROPERTY FOR A CLASS OF BLOCK OPERATOR MATRICES

Mitsuru Wilson
llmenau Technical university (Germany)

In this talk, we present a probed approximation of the spectrum of some classes of unbounded operators. The numerical range

$$W(T) := \{(T\,x, x) \mid x \in \mathcal{D}(T), \|x\| = 1\}.$$

and the quadratic numerical range

$$W^2\,(\mathcal{A}) := \bigcup \{\sigma_p(\mathcal{A}_{f,g}) | (f,g)^T \in \mathcal{D}(\mathcal{A}), \|f\| = \|g\| = 1\}$$

are supersets of the spectrum of an operator. In the present talk, we consider the following framework. Suppose H_1, H_2 are Hilbert spaces and

$$\mathcal{A}_{\pm D} = \begin{bmatrix} 0 & B \\ -B^* & \pm D \end{bmatrix} : \mathcal{D}(-B^*) \oplus \mathcal{D}(B) \subset H_1 \oplus H_2 \to H_1 \oplus H_2.$$

We show that the approximate point spectrum of an unbounded operator of the above type under some assumptions is contained in the closure of the quadratic numerical range and that the quadratic numerical range is contained in the numerical range.

BOUNDARY VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS WITH IMPULSE DISCRETE MEMORY

AnarT. Assanova
Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan,

In this communication we consider the boundary value problem for partial differential equations of hyperbolic type with impulse discrete memory in the following form:

$$\frac{\partial^2 v}{\partial t \partial x} = A(t,x)\frac{\partial v}{\partial x} + B(t,x)\frac{\partial v}{\partial t} + C(t,x)v +$$

$$+ A_0(t,x)\frac{\partial v(\gamma(t),x)}{\partial x} + B_0(t,x)\frac{\partial v(t,x)}{\partial t}\Big|_{t=\gamma(t)} + C_0(t,x)v(\gamma(t),x) + f(t,x), \quad t \neq \theta_j, \quad j = \overline{1, m-1}, \quad (1)$$

$$P(x)v(0,x) + S(x)v(T,x) = \varphi(x), \qquad x \in [0,\omega], \quad (2)$$

$$\lim_{t \to \theta_j + 0} v(t,x) - \lim_{t \to \theta_j - 0} v(t,x) = \varphi_j(x), \qquad x \in [0,\omega], \qquad j = \overline{1,m}, \quad (3)$$

$$v(t,0) = \psi(t), \qquad t \in [0,T], \quad (4)$$

where the $v(t,x) = (v_1(t,x), v_2(t,x), \ldots, v_n(t,x))$ is unknown vector function, the $(n \times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $A_0(t,x)$, $B_0(t,x)$, $C_0(t,x)$ and the n vector function $f(t,x)$ are piecewise continuous on $\Omega = [0,T] \times [0,\omega]$, the $(n \times n)$ matrices $P(x)$, $S(x)$ and the n vector function $\varphi(x)$ are continuously differentiable on $[0,\omega]$, $\gamma(t) = \xi_r$, $t \in [\theta_{r-1}, \theta_r)$, $\theta_{r-1} \le \xi_r < \theta_r$, $r = \overline{1,m}$, $0 = \theta_0 < \theta_1 < \ldots < \theta_m = T$, the n vector functions $\varphi_j(x)$ are continuously differentiable on

$[0, \omega]$, $j = \overline{1, m}$, the n vector function $\psi(t)$ is piecewise continuously differentiable on $[0, T]$. Partial differential equations of hyperbolic type with impulse discrete memory have intensively studied in recent decades [1]. These equations are also called partial differential equations with generalized piecewise-constant argument [2-5].

We study a questions for a uniqueness and existence of solution to problem (1)-(4). A constructive approach to solve the problem (1)-(4) is proposed by using methods and results in [6-7].

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SINGULAR BOUNDARY VALUE PROBLEMS FOR ODES AND THEIR APPROXIMATION

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Differential equations with singularities at the endpoints of the domain interval are often encountered in applications. In order to study the behavior of solutions at singular points, one can use so-called "limit solutions". We define the concept of a limit solution at singular points and prove that under certain assumptions on the right-hand part of the equation, the limit solution $x_0(t)$ possesses an attracting property; i.e. there exists a functional ball centered at $x_0(t)$ where the differential equation has at least one solution, and all solutions from this ball coincide with $x_0(t)$. We then construct approximating regular two-point boundary value problems that allow us find approximate solutions of the singular boundary value problem with any specified accuracy.

SHIFT-INVARIANT SPACES OF PERIODIC FUNCTIONS

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One of the basic ideas of multiresolution and wavelet analysis is the investigation of shift-invariant function spaces. In this talk one-dimensional shift-invariant spaces of periodic functions are generalized to multivariate shift-invariant spaces on non-tensor product patterns. Here, we discuss matrix shifts, where M is a $(d \times d)$-integer matrix with $\det M > 1$. Accordingly, an element of the shift-invariant space spanned by a function ϕ is given as the linear combination

$$\sum_{\ell \epsilon \Gamma} C_\ell \varphi(\cdot - 2\pi M^{-1}\ell),$$

where Γ denotes the full collection of coset representatives of $\mathbb{Z}^d/\Gamma\mathbb{Z}^d$. Decompositions of shift-invariant spaces are given by divisibility considerations.

For these spaces we discuss the dimension and we construct interpolatory and orthonormal bases. Possible patterns are classified. The results are applied to construct trigonometric polynomial shearlets which are special cases of directional de la Vallée Poussin type wavelets to detect singularities along curves of periodic bivariate functions.

This is joint work with R. Bergmann (Trondheim), D. Langemann (Braunschweig), K. Schober (Lübeck) and S. Stasyuk (Kiev).

THE NUMERICAL SOLUTION OF THE 3D-POISSON EQUATION FOR APPLICATIONS IN ACCELERATOR PHYSICS

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Numerical techniques in the field of particle accelerators are mainly driven by the design of next-generation accelerators: The need for higher simulation complexity and the necessity for more and more specialized algorithms arises from the ever increasing need to include a broader range of physical effects and geometrical details in a computer simulation.

Hereby, precise and fast 3D space charge calculations for bunches of charged particles are of growing importance in recent accelerator designs. One of the possible approaches is the particle mesh method, computing the potential of the bunch in the rest frame by means of Poisson's equation. The challenge lies in the development of efficient discretization and solution techniques considering a precise model of a tiny particle bunch as well as the geometry of the big beam pipe.

In this paper, we present numerical methods developed for high brightness electron beams provided by linac-based accelerators. These accelerators require several measures to preserve their high quality and to avoid instabilities, where the mitigation of the impact of residual ions is one of these measures. Our numerical results show the behavior of ions generated by electron bunch passages within the field of electrodes. The objective is to investigate the ion motion towards the electrodes and to study under which circumstances, equilibrium between ion generation and ion-clearing is established.

THE STOCHASTIC WAVE EQUATION – WHY AND HOW TO MODEL RANDOM FLUCTUATIONS

Lena Schadow
Universität zu Lübeck, Lübeck, Germany

The extensions of partial differential equations to stochastic partial differential equations gained importance in different fields of applications, such as neuroscience, financial mathematics, physics and biology. In this talk a brief introduction to stochastic partial differential equations with focus on the stochastic wave equation with additive white noise will be given. The stochastic wave equation extends the deterministic wave equation by incorporating stochasticity, thus enabling the representation of dynamic processes influenced by random fluctuations. We point out some research challenges while studying stochastic evolution equations and present the Itˆo formula as the link between deterministic calculus and stochastic processes.

SCATTERING ALONG A CURVE IN A PLAIN

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Scattering of a non-relativistic particle transversally bounded to a smooth asymptotically flat curve in the plane is considered. The curve is supporting the delta-like attractive interaction. The energy of the particle belongs to the negative part of the absolutely continuous spectrum. The existence of the wave operators for the scattering along the curve is proved.

SPECTRAL AND TORSION GEOMETRY OF QUANTUM GRAPHS

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Quantum graphs are an important example of metric measure spaces: they consist of finite collections of intervals glued at their endpoints (vertices) in a graph-like fashion. Under appropriate transmission conditions in the vertices, the heat equation is well-posed and may enjoy additional properties: most notably, conservation of mass. It turns out that the solutions converge to equilibrium at a rate that is given by the first non-trivial eigenvalue of the relevant realization of the Laplacian. It has been known since the 1980s that this quantity mirrors the connectivity and further combinatorial features of metric graphs. After offering an invitation to the lively topic of spectral geometry of quantum graphs, I will present a new quantity that has recently gained popularity: the torsional rigidity of a quantum graph, a quantity -- routinely considered for Dirichlet Laplacians on bounded planar domains -- whose definition goes back to Pólya. Its interesting geometric properties as well as its interplay with spectral theory have been discovered very recently: I will present an overview of the main result in this field.

This is based on joint work with Marvin Plümer and Sedef Özcan.

SPECTRAL METHODS IN THE THEORY OF P-ADIC NONLINEAR NONLOCAL PSEUDO-DIFFERENTIAL EQUATIONS

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The p-adic Mathematics is widely used in the Theoretical Physics and Biology. It attracts a great deal of interest in quantum mechanics, string theory, quantum gravity, spin-glass theory and system biology.

The concept of a hierarchical energy landscape is very important from the point of view of the description of relaxation phenomena in complex systems, in particular, glasses, clusters and proteins. This concept can be outlined as follows. A complex system is assumed to have a large number of metastable configurations which realize local minima on the potential energy surface. The local minima are clustered in hierarchically nested basins of minima, namely, each large basin consists of smaller basins, each of these consisting of even smaller ones, and so on. Thus we may say that the hierarchy of basins possesses ultrametric geometry and transitions between the basins determine the rearrangements of the system configuration for different time scales. Thus the key points of the concept of a hierarchical structure which is typical for p-adic world is the main advantage which can be used for the description of the complex phenomenon.

This talk gives a brief overview of the results related to the theory of pseudo-differential equations in the spaces of test and generalized functions on the field of p-adic numbers. Results related to the modern nonlinear theory of p-adic porous media equations and their solvability.

SLIGHTLY MODIFIED ORTHOGONALITY RELATION FOR THE JACOBI POLYNOMIALS AND EXACTLY SOLVABLE MODEL OF THE COMPLETELY POSITIVE OSCILLATOR-SHAPED QUANTUM WELL

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We are discussing some details of a slight modification of the orthogonality relation for the Jacobi polynomials in the interval $'-1 < x < 1'$ to the orthogonality relation for the shifted Jacobi polynomials in the interval $'a < x < b'$, where both $'a'$ and $'b'$ are the positive real parameters. Then, we apply this modification technique to the harmonic oscillator-shaped quantum well model completely defined within the positive values of the position. We managed to solve this problem exactly and obtained wavefunctions of the stationary states in terms of the shifted Jacobi polynomials as well as its non-linear energy spectrum. Some new limit relations between the Jacobi and the generalized Laguerre polynomials are also discussed.

This is the joint work with S.M. Nagiyev (Institute of Physics, Baku).

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ON THE SOLVABILITY OF PROBLEMS FOR THE LOADED INTEGRO-DIFFERENTIAL EQUATION INVOLVING A RIEMANN–LIOUVILLE FRACTIONAL INTEGRAL OPERATOR

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In recent years, in connection with intensive research on problems in mathematical physics, mathematical biology, problems of optimal control of the agro economical system, it has become necessary to investigate loaded equations. We will discuss how the boundary value problems for the loaded differential equations associated with nonlocal boundary value problems for classical partial differential equations. We are investigating the unique solvability of the problem for a loaded integro-differential equation.

ROLE OF THE CRITICAL EXPONENTS IN THE PROBLEM NONLINEAR HEAT CONDUCTIVITY WITH VARIABLE DENSITY AND TIME DEPENDENT DAMPING

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This work devoted to the role of critical exponents in a nonlinear parabolic equation exponent in the problem nonlinear heat conductivity with variable density and time dependent damping described by following Cauchy problem $Q=\{(t,x): t>0, x\in R^N\}$

$$Lu \equiv -|x|^{-n}\frac{\partial u}{\partial t}+\nabla(|x|^q u^{m-1}|\nabla u^k|^{p-2}\nabla u)+\varepsilon|x|^{-n}\gamma(\mathrm{t})|\nabla u^m|^{p_1}u^{q_1}=0,\ \varepsilon=\pm 1 \quad (1)$$

$$u|_{t=0}=u_0(x)\ge 0, x\in R^N \quad (2)$$

where $m,k\ge 1, p\ge 2$ are the given numerical parameters characterizing the nonlinear medium, $n,q>0$ means variable density, numbers $q_1, p_1,$ power of time depending damping term $\gamma(\mathrm{t})|\nabla u^m|^{p_1}u^{q_1}, 0<\gamma(\mathrm{t})\in C(0,\infty)$.

The equation (1) describes the processes of heat conductivity, diffusion, biological population, filtration in liquid and gas, spread of viruses and other different processes under action of time depending damping. Especially feathers of this problem is degenerating of equation (1). Because it in the domain where $u=0$ or $\nabla u=0$ may not have a solution in the classical sense. Therefore, in this case it is necessary to study the weak solutions with properties $0\le u, |x|^q u^{m-1}|\nabla u^k|^{p-2}\nabla u\in C(Q)$, and satisfy to the some integral identity. The problem (1), (2) for different values of numerical parameters intensively studied by lot of authors (see [1,2] and literature therein)

In this work based on self-similar analysis of numerical parameters the condition of global solvability of Fujita type, nonlinear property a finite speed of perturbation, a space localization, an estimate of weak solution for a slowly diffusion k(p-2)+m-1>0, quenching solution in a fast diffusion k(p-2)+m-1>0 and a critical cases when p>n and singular case p=n considered. The estimate of solution for each case established, an asymptotic behavior of the different type solution of a self-similar equation of the equation (1) proved. The problem choosing the initial approximation keeping nonlinear properties of solutions for numerical analysis of solution for different values of numerical parameters solved. Results of numerical experiments discussed.

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A MODIFICATION OF THE EULER POLYGONAL METHOD FOR SOLVING SEMI-PERIODICAL BOUNDARY VALUE PROBLEM FOR LOADED NONLINEAR HYPERBOLIC EQUATION

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On $\overline{\Omega} = [0;\ \omega] \times [0;\ T]$ we consider the following boundary value problem for loaded nonlinear hyperbolic equations with mixed derivatives:

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u(x,t)}{\partial x} + A_0(x,t)\frac{\partial u(x,t)}{\partial t}\Big|_{x=x_0} + f(x,t,u(x,t),\ \frac{\partial u(x,t)}{\partial t}), \tag{1}$$

$$u(x,0) = u(x,T), \qquad x \in [0,\omega] \tag{2}$$

$$u(0,t) = \psi(t), \quad t \in [0,T] \tag{3}$$

where $f: \overline{\Omega},\ \times R^2\ \to R$ continuous on $\overline{\Omega}$, $\psi(t)$ is continuously differentiable on $[0,T]$ and satisfies the condition $\psi(0) = \psi(T)$ functions. Functions $A(x,t), A_0(x,t)$ are continuous on $\overline{\Omega}$, x_0 is the load point. Problems for loaded hyperbolic equations have acquired particular relevance in connection with the study of the vibration stability of the wings of an aircraft loaded with masses, and in calculating the natural oscillations of antennas loaded with lumped capacitances and self-inductions [1]. Questions of existence and uniqueness of solutions to various problems for loaded partial differential equations of hyperbolic type and methods for finding their solutions are intensively studied by many authors [2-3].

A function $u(x,t) \in C(\overline{\Omega})$ which has a partial derivatives $\frac{\partial u(x,t)}{\partial x} \in C(\overline{\Omega})$, $\frac{\partial u(x_0,t)}{\partial x} \in C(\overline{\Omega})$, $\frac{\partial u(x,t)}{\partial t} \in C(\overline{\Omega})$, $\frac{\partial^2 u(x,t)}{\partial x \partial t} \in C(\overline{\Omega})$, called a classical solution of problem (1)-(3), if it satisfies equation (1) for all $(x,t) \in \overline{\Omega}$ and boundary value conditions (2)-(3).

We will be introduced new unknown functions $v(x,t) = \frac{\partial u}{\partial x}$, $w(x,t) = \frac{\partial u}{\partial t}$, $v^{x_0}(t) = \frac{\partial u}{\partial x}\Big|_{x=x_0}$ and then we has the equivalent problem

$$\frac{\partial v}{\partial t} = A(x,t)\mathrm{v(x,t)} + A_0(x,t)v^{x_0}(t) + f\big(x,t,u(x,t),\mathrm{w}(x,t)\big) \tag{4}$$

$$v(x,0) = v(x,T), \quad x,\ \ x_0 \in [0,w] \tag{5}$$

$$u(x,t) = \psi(t) + \int_0^x v(\xi,t)dt,\ \ w(x,t) = \dot{\psi}(t) \tag{6}$$

A modification of the Euler polygonal method is used to find the initial approximate solution of problem (4)-(6) and, based on this method, sufficient conditions for the existence of an isolated solution are established [4]. And also when applying a modification of the Euler polygonal method to problem (4)-(6), we obtain a family of boundary value problems for nonlinear loaded hyperbolic equations. To find their numerical solutions we used the parametrization method [3]. The parametrization method gives not only a numerical solution, also makes it possible to obtain the conditions for the existence of families of boundary value problems.

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A COMPUTATIONAL METHOD FOR SOLVING PROBLEM FOR IMPULSIVE DIFFERENTIAL EQUATIONS WITH LOADINGS

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We consider the following linear two-point boundary value problem for impulsive differential equations with loadings:

$$\frac{dx}{dt} = A_0(t)x + \sum_{i=1}^{m} M_i(t) \lim_{t\to\theta_i+0} \dot{x}(t) + \\ + \sum_{i=1}^{m} K_i(t) \lim_{t\to\theta_i+0} x(t) + f(t), \quad t \in (0,T)\backslash\{\theta_1, \theta_2, \dots, \theta_m\}, \tag{1}$$

$$B_0 x(0) + C_0 x(T) = d, \quad d \in R^n, \quad x \in R^n, \tag{2}$$

$$B_i \lim_{t\to\theta_i-0} x(t) - C_i \lim_{t\to\theta_i+0} x(t) = \varphi_i, \quad \varphi_i \in R^n, \ \ i = \overline{1,m}. \tag{3}$$

Here $(n\times n)$-matrices $A_0(t),\ M_i(t),\ K_i(t), (i=\overline{1,m})$, and n -vector-function $f(t)$ are piecewise continuous on $[0,T]$ with possible discontinuities of the first kind at the points $t=\theta_i$, $(i=\overline{1,m})$; B_i and C_i , $(i=\overline{0,m})$ are constant $(n\times n)$-matrices, and φ_i , $(i=\overline{1,m})$ and d are constant n vectors, $0=\theta_0<\theta_1<\dots<\theta_m<\theta_{m+1}=T$.

Many important problems for impulsive differential equations with loadings and methods for finding their solutions are considered in [1], [2]. In the present paper, linear two-point boundary value problem for impulsive differential equations with loadings is investigated by the Dzhumabaev parameterization method [3]. A computational method is offered to solve the problem (1) - (3).

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ON THE SOLVABILITY OF THE PERIODIC BOUNDARY VALUE PROBLEM FOR DELAY DIFFERENTIAL EQUATION

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A periodic boundary value problem for a system of delay differential equations is considered on the interval $(0,T)$

$$\frac{dx(t)}{dt} = A(t)x(t) + B(t)x(t-\tau) + f(t), \qquad x \in R^n, \qquad \tau > 0, \tag{1}$$

$$x(t) = diag[x(0)] \cdot \varphi(t), \qquad t \in [-\tau, 0\,], \tag{2}$$

$$x(0) = x(T). \tag{3}$$

Here the $n \times n$ matrices $A(t), B(t)$ and the vector function $f(t)$ are continuous on $[0,T]$, $\varphi\colon [-\tau, 0] \to R^n$ is a continuously differentiable vector function such that $\varphi_i(0) = 1, i = \overline{1,n},\ \tau$ is a constant delay.

A solution to the boundary-value problem (1)-(3) is a continuous on $[-\tau, T]$, continuously differentiable on $(0,T)$ vector function $x^*(t)$, satisfying Eq. (1) and conditions (2), (3).

To solve the problem (1)-(3), the idea of the parameterization method [1] is used, namely: the interval at which the problem is being considered is divided into subintervals whose lengths do not exceed the values of the constant delay; constant parameters are introduced at the left ends of these intervals; a new unknown function is introduced at each subinterval. Thus, the problem (1)-(3) under consideration is reduced to an equivalent multipoint boundary value problem for delay differential equations. On each of the subintervals, the auxiliary Cauchy problems without delay with zero initial conditions at the left ends of the subintervals are considered sequentially. We propose an algorithm for finding the solution of a multipoint boundary value problem for delay differential equations with containing parameters. At each step of the algorithm a system of linear algebraic equations is solved to determine the values of the parameters, and an analogue of the Cauchy formula is used to obtain solutions to auxiliary Cauchy problems.

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ONE APPROACH TO FIND AN APPROXIMATE AND A NUMERICAL SOLUTION OF A NONLINEAR BOUNDARY VALUE PROBLEM

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We consider the boundary value problem for Fredholm integro-differential equation with a nonlinear differential part

$$\frac{dx}{dt} = f(t,x) + \sum_{k=1}^{m} \varphi_k(t) \int_0^T \psi_k(\tau)x(\tau)d\tau, \quad t \in (0,T), \ \ x \in R^n, \tag{1}$$

$$g\big(x(0), x(T)\big) = 0, \tag{2}$$

where $f: [0,T] \times R^n \to R^n$ is continuous function, the $n \times n$ matrices $\varphi_k(t), \psi_k(\tau), k = \overline{1,m}$, are continuous on $[0,T]$, $\|x\| = \max\limits_{i=\overline{1,n}} |x_i|$.

Denote by $C([0,T], R^n)$ the space of all continuous functions $x: [0,T] \to R^n$ with the norm $\|x\|_1 = \max\limits_{t\in[0,T]} \|x(t)\|$.

By a solution to problem (1), (2) we mean a continuously differentiable function $x(t) \in C([0,T], R^n)$ on $(0,T)$ that satisfies equation (1) and boundary condition (2). It is assumed that the considered functions at the end points of the intervals have one-sided derivatives.

The aim of the communication is to develop a constructive method of solving boundary value problem (1), (2). To this end, we use the parameterization method [1] and results of [2-6].

We propose a method to solve boundary value problem (1), (2). By introducing additional parameters chosen as the values of the solution at the left-end points of the partition subintervals, the problem under consideration is transformed into an equivalent boundary value problem for a system of nonlinear integro-differential equations with parameters on the subintervals. For fixed parameters, we obtain a special Cauchy problem for this system, which is represented as a nonlinear operator equation and solved by an iterative method. By substitution of the solution to the special Cauchy problem into the boundary condition and the continuity conditions of the solution to the original problem at the interior partition points, we construct a system of nonlinear algebraic equations in parameters. It is proved that the solvability of this system provides the existence of a solution to the original boundary value problem. The algorithm for solving the special Cauchy problem includes two auxiliary problems: the Cauchy problems for ordinary differential equations and the evaluation of definite integrals. The accuracy of the method that we propose to solve the boundary value problem depends on the accuracy of methods applied to the auxiliary problems and does not depend on the number of the partition subintervals. Since iterative methods are used to solve both the constructed system of algebraic equations and the special Cauchy problem, we offer an approach to find an initial guess for the solutions to these problems.

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CREATION OF DECISIONS IN A BROAD SENSE THE SYSTEMS OF THE EQUATIONS IN PRIVATE DERIVATIVES OF THE FIRST ORDER WITH PERIODIC CONDITIONS

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Annotation: Sufficient conditions of living and uniqueness of the solution in a broad sense of the systems of the equations in private derivatives of the first order with periodic conditions are received.
Keywords: the differential equations in private derivatives, classical solution, vector-function, decisions in a broad sense.

The differential equations in private derivatives often model various applied problems of hydroaeromechanics, chemical kinetics, the theory of catalytic reactions, etc.

In classical solutions of nonlinear equations with first-order partial derivatives, even with arbitrarily smooth initial functions, features can form with time growth. Therefore, there is an urgent need to expand the concept of classical solutions to first-order partial differential equation systems.

Decisions in a broad sense (according to Friedrichs) the quasilinear hyperbolic systems of the equations in private derivatives with two independent variables are constructed in works [1-3].

Is considered the equations in private derivatives of the first order

$$\frac{\partial x}{\partial t}+\sum_{j=1}^{m}a_j(t,\varphi,\psi)\frac{\partial x}{\partial \varphi_j}+\sum_{j=1}^{k}b_j(t,\varphi,\psi)\frac{\partial x}{\partial \psi_j}=P(t,\varphi,\psi)x+f(t,\varphi,\psi) \qquad (1)$$

where $t\in(-\infty,+\infty)=R,\quad \varphi=(\varphi_1,...,\varphi_m)\in R^m,\quad \psi=(\psi_1,...,\psi_k)\in R^k$, $a(t,\varphi,\psi),\ \ b(t,\varphi,\psi)$ – continuous vector-functions of dimension, respectively, having periodicity, smoothness properties

$$a(t+\theta,\varphi+q\omega,\psi)=a(t,\varphi,\psi)\in C^{(0,1,1)}_{t,\varphi,\psi}(R\times R^m\times R^k),$$
$$b(t+\theta,\varphi+q\omega,\psi)=b(t,\varphi,\psi)\in C^{(0,1,1)}_{t,\varphi,\psi}(R\times R^m\times R^k), \tag{2}$$

and bounds with a norm maximizing the Euclidean metric vector- functions

$$\|a\|\le\alpha_0,\ \left\|\frac{\partial}{\partial\varphi}a\right\|\le\alpha_1,\quad \left\|\frac{\partial}{\partial\psi}a\right\|\le\alpha_2,$$
$$\|b\|\le\beta_0,\ \left\|\frac{\partial}{\partial\varphi}b\right\|\le\beta_1,\quad \left\|\frac{\partial}{\partial\psi}b\right\|\le\beta_2, \tag{3}$$

for all $q=(q_1,...,q_m)\in Z^m$, Z – a set of integers. Periods $\omega_0=\theta,\ \omega_1,...,\omega_m$ – positive incommensurable constants, $q\omega=(q_1\omega_1,q_2\omega_2,...,q_m\omega_m)$ – a vector of the multiple periods, $\omega=(\omega_1,...,\omega_m)$, α_0,β_0, α_1,α_2, β_1,β_2 – some positive constants. $x=(x_1,...,x_{n_1})$ – search vector-function, $P(t,\varphi,\psi)$ $-n_1\times n_1-$ the matrix having properties of frequency as regards variables and limitation:
$\|P\|\le k_0=const>0$,

$$P(t+\theta,\varphi+q\omega,\psi)=P(t,\varphi,\psi)\in C(R\times R^m\times R^k),\ q\in Z^m \tag{4}$$

$f(t,\varphi,\psi)$ - n_1 – vector function, having properties of frequency

$$f(t+\theta,\varphi+q\omega,\psi)=f(t,\varphi,\psi)\in C(R\times R^m\times R^k),\ q\in Z^m \tag{5}$$

and limitations

$$\|f\|\le K=const>0. \tag{6}$$

We will consider the equation (1) along characteristic $\{\lambda(t,t_0,\varphi_0,\psi_0),\xi(t,t_0,\varphi_0,\psi_0)\}$, then we will receive the system of the ordinary differential equations:

$$\frac{d\tilde{x}}{dt}=\tilde{P}\tilde{x}+\tilde{f} \tag{7}$$

where $\tilde{x}=x(t,\lambda(t,t_0,\varphi_0,\psi_0),\xi(t,t_0,\varphi_0,\psi_0))$, $\tilde{P}=P(t,\lambda(t,t_0,\varphi_0,\psi_0),\xi(t,t_0,\varphi_0,\psi_0))$,

$\tilde{f}=f(t,\lambda(t,t_0,\varphi_0,\psi_0),\xi(t,t_0,\varphi_0,\psi_0))$.

$$x^*(t,\varphi,\psi)=\int_{-\infty}^{t}X\ (t,\varphi,\psi,s,\lambda(s,t,\varphi,\psi),\xi(s,t,\varphi,\psi))\times$$
$$\times f(s,\lambda(s,t,\varphi,\psi),\xi(s,t,\varphi,\psi))ds \tag{8}$$

Theorem. Let conditions (2) to (6) be met. Then the linear heterogeneous system (1) has a single multi-period solution in the broad sense (8).

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INVERSE SOURCE PROBLEMS WITH INTEGRAL OVERDETERMINATION FOR THE SUBDIFFUSION EQUATIONS ON METRIC GRAPHS

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We investigate the inverse source problems with the integral overdetermination condition for the subdiffusion equation on the metric star graph in Sobolev spaces. The existence and uniqueness of the strong solution to the direct problem were proven with the functional method based on a priori estimates. The unique solvability of the inverse problem in the Sobolev space is proven. For that, we reduce the inverse source problem to the operator-based equation and prove that the corresponding resolvent operator is well-defined and continuous.

METHOD OF FUNCTIONAL PARAMETRIZATION FOR SOLVING A SEMI-PERIODIC INITIAL PROBLEM FOR FOURTH-ORDER PARTIAL DIFFERENTIAL EQUATIONS

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A semi-periodic initial boundary-value problem for a fourth-order system of partial differential equations is considered. Using the method of functional parametrization, an additional parameter is carried out and the studied problem is reduced to the equivalent semi-periodic problem for a system of integro-differential equations of hyperbolic type second order with functional parameters and integral relations. An interrelation between the semi-periodic problem for the system of integro-differential equations of hyperbolic type and a family of Cauchy problems for a system of ordinary differential equations is established. Algorithms for finding of solutions to an equivalent problem are constructed and their convergence is proved. Sufficient conditions of a unique solvability to the semi-periodic initial boundary value problem for the fourth-order system of partial differential equations are obtained.

In the present paper, on the domain $\Omega = [0, T] \times [0, \omega]$ we consider the following semi-periodic initial boundary value problem for a fourth order system of partial differential equations

$$\frac{\partial^4 u}{\partial t^3 \partial x} = A_1(t,x)\frac{\partial^3 u}{\partial t^2 \partial x} + A_2(t,x)\frac{\partial^3 u}{\partial t^3} + A_3(t,x)\frac{\partial^2 u}{\partial t^2} + A_4(t,x)\frac{\partial^2 u}{\partial t \partial x} + \\ + A_5(t,x)\frac{\partial u}{\partial t} + A_6(t,x)\frac{\partial u}{\partial x} + A_7(t,x)u + f(t,x), \quad (1)$$

$$u(0,x) = \varphi_1(x), \ \ x \in [0,\omega], \tag{2}$$
$$\left.\frac{\partial u(t,x)}{\partial t}\right|_{t=0} = \varphi_2(x), \qquad x \in [0,\omega], \tag{3}$$
$$\left.\frac{\partial^2 u(t,x)}{\partial t^2}\right|_{t=0} = \left.\frac{\partial^2 u(t,x)}{\partial t^2}\right|_{t=T}, \qquad x \in [0,\omega], \tag{4}$$
$$u(t,0) = \psi(t), \ \ t \in [0,T], \tag{5}$$

where $u(t,x) = col(u_1(t,x),\ u_2(t,x), \dots, u_n(t,x))$ is unknown function, the $n \times n$ matrices $A_i(t,x)$, $(i = \overline{1,7})$ and n vector–function $f(t,x)$ are continuous on Ω; n vector–function $\psi(t)$ are continuously three times differentiable on $[0,T]$; the n vector–functions $\varphi_1(x)$ and $\varphi_2(x)$are continuously differentiable on $[0,\omega]$.

Theorem 2. Suppose that for some m, $m = 1,2,3, \dots$, the $n \times n$ -matrix $D_m(T,x)$ is invertible for all $[0,\omega]$ and the inequalities

a) $\|[D_m(T,x)]^{-1}\| \le \gamma_m(T,x)$, and $\gamma_m(T,x)$ is a positive continuous function for all $x \in [0,\omega]$;

b) $q_m(T,x) = \gamma_m(T,x)\left\{e^{\alpha(x)T} - 1 - \alpha(x)T - \dots - \frac{1}{m!}[\alpha(x)T]^m\right\} \le \chi < 1$, where χ –is constant, $\alpha(x) = \max\limits_{t\in[0,T]}\|A_1(t,x), A_4(t,x), A_6(t,x)\|$.

Then there is a unique classical solution $u^*(t,x)$ to problem (1)–(5), defining from the following integral representation

$$u^*(t,x) = \varphi_1(x) + t \cdot \varphi_2(x) + \frac{t^2}{2}\cdot\lambda^*(x) + \int_0^t\int_0^\tau \tilde{w}^*(\tau_1,x)d\tau_1 d\tau, \qquad (t,x) \in \Omega.$$

CONSTRUCTION OF THE LAGRANGE EQUATION BY A GIVEN KINEMATIC FORM OF THE NEWTON EQUATION IN THE PRESENCE OF RANDOM PERTURBATIONS

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Given second-order Ito stochastic equations, we construct almost surely equivalent stochastic equations of the Lagrangian structure. We establish the conditions for the direct and indirect analytical representations of the Lagrangian in the presence of random perturbations.

Definition. The kinematic form of the Newton equation in the presence of random perturbations is the equation of the form

$$d\dot{x}_\nu = F_\nu(x,\dot{x},t)dt + \sigma_{\nu j}(x,\dot{x},t)d_0\xi^j, \ \left(\nu = \overline{1,n}, \ \ j = \overline{1,r}\right) \tag{1}$$

and, accordingly, the main form of the Newton's equation in the presence of random perturbations is the equation of the form

$$A_{\nu i}(x,\dot{x},t)d\dot{x}_i + B_\nu(x,\dot{x},t)dt = \sigma_{\nu j}(x,\dot{x},t)d_0\xi^j, \left(\nu = \overline{1,n}, \quad j = \overline{1,r}\right). \tag{2}$$

The equation of the form

$$d\left(\frac{\partial L}{\partial \dot{x}_k}\right) - \frac{\partial L}{\partial x_k}dt = \sigma'_{kj}(x,\dot{x},t)d_0\xi^j, \left(k = \overline{1,n}, \quad j = \overline{1,r}\right) \tag{3}$$

is called the stochastic equation of the Lagrangian structure.

We assume that all functions in the above equations are smooth enough for further reasoning and satisfy the existence and uniqueness theorem for the solution of the Cauchy problem in the class of Itô stochastic differential equations.

Given the kinematic form (1) of the Newton equations in the presence of random perturbations, we pose the problem of constructing equivalent stochastic equations with the Lagrangian structure of the form (3). In other words, by the given functions F_ν and $\sigma_{\nu j}$ in (1), it is required to determine the conditions on L and σ'_{kj} under which Eq. (1) is equivalent to Eq. (3).

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TUNNELING IN SOFT WAVEGUIDES: CLOSING A BOOK

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In this talk, we present recent new results in the field of quantum waveguides - Schrödinger operators in curved regions - with 'soft' walls. We investigate the properties of their spectrum in two dimensions in the setting of the generalized ‘bookcover’ shape, that is, Schrödinger operator with the potential in the form of a ditch consisting of a finite curved part and straight asymptotes which are parallel or almost parallel pointing in the same direction.

We show how the eigenvalues accumulate when the angle between the asymptotes tends to zero. In case of parallel asymptotes the existence of a discrete spectrum depends on the ditch profile. We prove that it is absent in the weak-coupling case, on the other hand, it exists provided the transverse potential is strong enough. We also present a numerical example in which the critical strength can be assessed.

TO THE THEORY OF THE OVERDETERMINATED SYSTEM OF VOLTERRA-TYPE THREE INTEGRAL EQUATIONS WITH SINGULAR BOUNDARY LINES

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In this paper, we study an overdetermined system of Volterra-type three integral equations with singular lines, which consists of a two-dimensional Volterra-type integral equation with two singular lines and two one-dimensional Volterra-type integral equations with a singular line of the form

$$\begin{cases} u(x,y)+\lambda\int\limits_a^x \frac{u(t,y)}{t-a}dt-\mu\int\limits_y^b \frac{u(x,s)}{b-s}ds+\delta\int\limits_a^x \frac{dt}{t-a}\int\limits_y^b \frac{u(t,s)}{b-s}ds=f(x,y) \\ u(x,y)+\gamma\int\limits_0^a \frac{u(t,y)}{t-a}dt=g_1(x,y) \\ u(x,y)-\chi\int\limits_y^b \frac{u(x,s)}{b-s}ds=g_2(x,y) \end{cases}$$

in a rectangle D={$a < x < a_0, b_0 < y < b$}. The conditions for the compatibility of the equations of the system for all possible values of the parameters of the equations of the system are determined. It has been established that when the parameters of the two-dimensional equation of the integral equation of the system are interconnected in a special way, depending on the signs of the parameters, the explicit solution of the system of equations may contain an arbitrary function, an arbitrary constant, or the solution of the system of equations may be unique.

THE ORDER 3 SYMMETRY IN THE WEYL-HEISENBERG GROUP

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The problem of existence of symmetric informationally-complete positive operator-valued measures (SICs for short) in every dimension is known as Zauner's conjecture and is open to this day. Most of the known SIC examples are constructed as an orbit of the Weyl-Heisenberg group action. It appears that in these cases the SIC is invariant under an order 3 symmetry that acts on the Weyl-Heisenberg group, the fact that has no explanation either. Here we give an overview of those order 3 symmetries and show how they appear naturally in representations of the triangle group (3,3,3).

ON THE DISCRETE SPECTRUM OF THE SCHRÖDINGER OPERATOR OF THE SYSTEM OF THREE-PARTICLES WITH MASSES $m_1 = m_2 = \infty$ AND $m_3 < \infty$ ON THE THREE DIMENSIONAL LATTICE

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In this paper, we study the spectrum of the discrete Schrödinger operator H corresponding to a system of three particles, with masses $m_1 = m_2 = \infty$ and $m_3 < \infty$, and interacting via short-range pair potentials in the three dimensional lattice. Using the integral decomposition, the study of spectrum of the operator H is reduced to investigating spectra of the more convenient fiber operators $H(K)$, depending on the quasi-momentum K. The essential spectrum $\sigma_{ess}(H(K))$ of $H(K)$ is the union of the spectra of the channel operators $H_\alpha(K), \alpha = 1,2,3$ (see, [1, 2]). We prove that the discrete spectrum of the corresponding Schrödinger operator is infinite (Theorem 1). Moreover, we show that an infinitely many eigenvalues may appear in the gap of the essential spectrum for some values of the interaction energy. In the one-dimensional case, a similar problem was considered in [3] and [4], and infiniteness of the discrete spectrum was proven.

The operator $H(K)$, $K \in \mathbb{T}^3 = (-\pi, \pi]^3$ associated with the current system is of the form

$$H(K) = H_0(K) - V_{12} - V_{23} - V_{31}, \tag{1}$$

where the operators $H_0(K)$ and $V_{\beta\gamma}$ are defined on the Hilbert space $L_2((\mathbb{T}^3)^2)$ by

$$(H_0(K)f)(p,q) = E(K;p,q)f(p,q), \quad f \in L_2((\mathbb{T}^3)^2),$$

$$(V_1 f)(p,q) = \frac{\mu_1}{(2\pi)^3}\int\limits_{\mathbb{T}^3} f(p,t)dt, \quad (V_2 f)(p,q) = \frac{\mu_2}{(2\pi)^3}\int\limits_{\mathbb{T}^3} f(t,q)dt,$$

$$(V_3 f)(p,q) = \frac{\mu_3}{(2\pi)^3}\int\limits_{\mathbb{T}^3} f(t,p+q-t)dt, \quad f \in L_2((\mathbb{T}^3)^2),$$

where

$$E(K;p,q) = \varepsilon(K-p-q)/m_3,$$

and the real-valued continuous function

$$\varepsilon_\alpha(p) = \frac{1}{m_\alpha}\epsilon(p), \quad \epsilon(p) = \sum_{j=1}^{3}(1-\cos p^{(j)}), \quad p = (p^{(1)},\ldots,p^{(d)}) \in \mathbb{T}^3, \tag{2}$$

is called the *dispersion relation of the α-th normal mode* associated with the free particle α ($\alpha = 1,2,3$).

Here $\mu_\alpha \geq 0$, $\alpha = 1,2,3$ is called an energy of the interactions.

Let us define the following function

$$\Delta_\alpha(z) = 1 - \frac{\mu_\alpha}{(2\pi)^d}\int\limits_{\mathbb{T}^3}\frac{ds}{\varepsilon_3(s)-z}, \quad z \in \mathbb{C}\setminus[E_{\min}(K), E_{\max}(K)], \quad \alpha = 1,2.$$

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Set

$$\mu_\alpha^0 = \left(\frac{1}{(2\pi)^d} \int\limits_{\mathbb{T}^3} \frac{ds}{\varepsilon_3(s)} \right)^{-1}, \quad \alpha = 1, 2.$$

Lemma 1. *Let* $\alpha = 1, 2$.

(a) If $0 < \mu_\alpha \le \mu_\alpha^0$*, then* $\Delta_\alpha(z)$ *has no zeros in the interval* $(-\infty, E_{\min}(K))$.

(b) If $\mu_\alpha > \mu_\alpha^0$*, then* $\Delta_\alpha(z)$ *has a unique simple zero in the interval* $(-\infty, E_{\min}(K))$*, i.e.,* $\Delta_\alpha(z_\alpha^0) = 0$.

Lemma 2. *Let* $\mu_\alpha > 0, \alpha = 1, 2$*, and let* $\mu_3 \ge 0$*. Then, for every* $K \in \mathbb{T}^3$*, the following relations hold:*

For the essential spectrum of the main operator $H(K)$

$$\sigma_{ess}(H(K)) = [0, 6/m_3] \cup (\Lambda_1 \cup \Lambda_2 \cup [-\mu_3, 6/m_3 - \mu_3]),$$

where

$$\Lambda_\alpha = \begin{cases} \emptyset, & \textit{if} \quad 0 < \mu_\alpha \le \mu_0 \\ \{z_\alpha^0\}, & \textit{if} \quad 0 < \mu_\alpha \le \mu_0 \end{cases}, \quad \alpha = 1, 2.$$

Moreover, we have

$$\sigma_{ess}(H(K)) = (\Lambda_1 \cup \Lambda_2) \cup [-\mu_3, 6/m_3], \quad 0 \le \mu_3 \le 6/m_3,$$

$$\sigma_{ess}(H(K)) = (\Lambda_1 \cup \Lambda_2 \cup [-\mu_3, 6/m_3 - \mu_3]) \cup [0, 6/m_3], \quad \mu_3 > 6/m_3,$$

where $\{z_\alpha^0\} = \varnothing$ *if*

Theorem 1. *Let* $z_{\min} = \inf\{z_1^0, z_2^0\}$ *and let* $z_{\max} = \sup\{z_1^0, z_2^0\}$*. Fix arbitrary* μ_1, μ_2 *and* μ_3*. Then, there exist infinite sets of eigenvalues* $z_n \in (-\infty, z_{\min})$ *and* $\xi_n \in (z_{\max}, E_{\min}(K))$*,* $n \in \mathbb{Z}^3$*, of* $H(K)$ *such that*

$$\lim_{n\to\infty} z_n = z_{\min} \quad \textit{and} \quad \lim_{n\to\infty} \xi_n = z_{\max}.$$

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